Fat Tails, Exponents, and Extreme Uncertainty: Simulating Catastrophe in DICE

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Abstract

The problem of low-probability, catastrophic risk is increasingly central to discussion of climate science and policy. But the integrated assessment models (IAMs) of climate economics rarely incorporate this possibility. What modifications are needed to analyze catastrophic economic risks in an IAM? We explore this question using DICE, a well-known IAM. We examine the implications of a fat-tailed probability distribution for the climate sensitivity parameter, a focus of recent work by Martin Weitzman, and the shape of the damage function, one of the issues raised by the Stern Review. Forecasts of disastrous economic outcomes in DICE are easily produced by the interaction of these two innovations, but not by either one alone.

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1. Introduction

Economic assessment of climate change and climate policy depends on information that is not currently available, and may not become available until it is too late to do anything about it. Two central uncertainties, in particular, pose challenges to quantitative economic analysis. First, how bad will the climate get – that is, how much will temperatures rise as a result of increasing atmospheric concentrations of greenhouse gases? Second, how bad will the worsening climate be for the economy – that is, how much economic damage will be caused by increased temperatures and associated physical impacts of climate change?[1] Such questions remain unanswered and perhaps intrinsically unanswerable except in retrospect, despite the increasingly detailed understanding of climate processes that is emerging from scientific research. Yet with bad enough answers to these questions, climate change might lead to disastrous results for the global economy.

Conventional economic analysis does not appear to be stymied by the problems of irreducible uncertainty and catastrophic risks. Integrated assessment models (IAMs) often adopt deterministic estimates or "best guesses" about a number of crucial unknowns. This procedure eliminates uncertainty from the model, at the cost of making the results dependent on the particular estimates that are employed. (Ackerman *et al.* 2009a; Stanton *et al.* 2009). Using their chosen resolutions of key uncertainties, IAMs have often found that the optimal policy response is to do relatively little about climate change in the near term. The catastrophic risks that are increasingly discussed in climate science and policy analyses almost never translate into catastrophic economic outcomes in IAMs.

This paper explores what it would take to make DICE, one of the best-known IAMs, forecast an economic catastrophe. William Nordhaus, the creator of DICE, reports that his model does not appear to display extreme responses to uncertainties about key input parameters, and concludes that

> "…models such as [DICE] have limited utility in looking at the potential for catastrophic events." (Nordhaus 2008, p. 147)

"Catastrophe" can be interpreted in two ways, either as an abrupt discontinuity or as an unexpected, very bad outcome. As Nordhaus suggests, in the absence of hard scientific information about discontinuities, it is difficult to incorporate them into a deterministic model like DICE. (The probabilistic logic of the PAGE model is better suited to this task, as discussed below.) The other interpretation of catastrophe – things turning out really badly – is easier to model; the upbeat conclusion of the DICE default scenarios is not the only message that this model can convey. We offer a new way of looking at DICE, in which disastrous economic outcomes are natural results of plausible values for key uncertain parameters.

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[1] Climate damages result not only from increasing temperatures, but also from other physical changes such as rising sea levels, changes in precipitation, and increasing frequency of extreme weather events. Since these are all broadly correlated, growing more intense as greenhouse gas concentrations and temperatures rise, we use temperature as a numerical index of the severity of the physical impacts of climate change in general.

Two recent contributions to the economics of climate change have produced a richer understanding of the role of uncertainty. Martin Weitzman's theoretical analysis of "fat-tailed" probability distributions examines the uncertainty about the temperature increase that will result from rising greenhouse gas emissions (Weitzman 2007; 2009). The Stern Review (Stern 2006), among its other important points, explores the uncertainty about the shape of the damage function which relates economic impacts to temperature.

Each of these theoretical contributions highlights the role of a specific parameter used in IAMs. Weitzman's analysis addresses uncertainty about the climate sensitivity parameter, i.e. the long-term temperature change that will result from a doubling of atmospheric CO_2 concentrations. The Stern Review illustrates the importance of the "damage function exponent": global economic damages from climate change are often assumed to depend on the square of temperature, but could just as easily be tied to the cube or other power of temperature (measured as degrees above a pre-industrial or twentieth-century baseline).

How much would DICE outputs and recommendations be changed by variation in the climate sensitivity parameter and damage function exponent? DICE normally forecasts steady economic growth, even under the impacts of business-as-usual climate change. It finds that the optimal policy is a modest carbon tax, starting at $7.40 per ton of CO_2 today and rising only to $25 in 2050 and $55 in 2100 (Nordhaus 2008, pp. 14-16). That policy slows the growth of carbon emissions, but does not cause a reduction: while business-as-usual emissions grow by 166 percent during this century, emissions under the optimal tax regime grow by 52 percent (calculated from *ibid.*, p. 100).

Our results suggest that changing either the climate sensitivity parameter or the damage function exponent alone has only a limited effect on DICE's upbeat projections. Simultaneous changes in both parameters, however, can lead to a forecast of severe losses under business as usual, and an optimal policy of very rapid reduction in emissions. Thus the optimistic projections and modest optimal policies often attributed to models such as DICE may be artifacts of parameter choices, rather than robust forecasts about an uncertain future.

2. Catastrophic risk and damages in DICE

Like many IAMs, DICE is a deterministic model, using best guesses or expected values over a hypothesized probability distribution in order to address uncertainties about future costs and benefits (Stanton *et al.* 2009). In particular, DICE makes the common assumptions that the value of the climate sensitivity parameter is 3 (the best estimate according to IPCC 2007), and that global damages depend on the square of temperature increases.

DICE is one step ahead of a number of other IAMs in addressing uncertainty: it assumes that an abrupt loss of a significant share of world output could occur, with a probability that is low but rises with increasing temperatures (Nordhaus 2008). The magnitude of the catastrophe was initially derived from a survey of expert opinion in the early 1990s, and has since been revised upward as climate projections have become more ominous. The initial survey itself elicited a wide distribution of estimates, with a noticeable minority of forecasts of an enormous potential catastrophe (Roughgarden and Schneider 1999).

DICE, however, sidesteps uncertainty by calculating the expected value of low-probability, high-cost catastrophic damages. In DICE-2007, the expected value of a climate catastrophe is 1.2 percent of world output at 2.5°C of warming, and 4.7 percent at 6° (Nordhaus 2007a, p. 24). The expected value is then included in the calculation of damages that will predictably result from a given temperature increase. Thus DICE addresses catastrophic risk in theory, only to turn it into a deterministic guess in practice;[2] we describe it as a guess because there is very little empirical information available about the values of either the probability or the magnitude of the damages in question.

Letting climate damages as a fraction of world output be **d**, and temperature increase since a base year be **T**, it has become common to assume a simple power law, such as

$$(1) \qquad \mathbf{d} = \mathbf{aT^N}$$

DICE uses a slight variant, which is quite similar to (1) at low temperatures:

$$(2) \qquad \mathbf{d} = \mathbf{aT^N} / (1 + \mathbf{aT^N})$$

The use of (2) prevents climate damages from exceeding the value of world output; this would be a matter of common sense if damages could only reduce current income, as DICE assumes. If, more realistically, climate damages may also include the destruction of capital assets, then damages could exceed 100 percent of a year's output.

Nordhaus estimates that for a 2.5°C temperature increase from 1900, annual climate damages, including the expected value of a possible catastrophe, amount to just 1.77 percent of world output.[3] This represents net damages, combining benefits in some areas with costs in other areas: a relatively large monetary value is placed on subjective enjoyment of warmer temperatures, offsetting some but not all of the predicted damages in other areas. (The subjective enjoyment of warming played an even bigger role in the previous version of DICE, as discussed in Ackerman and Finlayson 2006; the same calculation is used in DICE-2007, but the new version does not allow global net benefits from warming.) On the assumption that **N** = 2 in (2), the Nordhaus estimate for damages at 2.5°C implies that **a** = .002838.

[2] Nordhaus presents a small-scale Monte Carlo analysis using the latest DICE in Nordhaus 2008, but this is external to the model – and is not central to his presentation of its results.
[3] The supporting documentation for DICE-2007 also offers an estimate that climate damages at 6°C would amount to a mere 8.23 percent of world output, but this number is barely explained, and the final form of the damage function does not appear to rely on it (Nordhaus 2007a, p. 24).

3. Fat tails and unbounded risks

Martin Weitzman (2007; 2009) has argued that the economic analysis of climate change is dominated by the problem of intrinsically limited information about potentially unbounded risks. Let the value of climate damages be $\mathbf{D}(\mathbf{x})$, where $\mathbf{x}$ is the climate sensitivity parameter, and let $\mathbf{p}(\mathbf{x})$ be the probability distribution of $\mathbf{x}$. As $\mathbf{x}$ increases, $\mathbf{D}(\mathbf{x})$ also increases, with no obvious upper limit. The expected value of climate damages is

$$(3) \qquad E[\mathbf{D}(\mathbf{x})] = \int \mathbf{D}(\mathbf{x})\mathbf{p}(\mathbf{x})\, d\mathbf{x}$$

If there is a sufficiently large body of empirical evidence about $\mathbf{x}$, then the best estimate of $\mathbf{p}(\mathbf{x})$ might be a normal distribution or other "thin-tailed" distribution – that is, a distribution which is known to have low probabilities of extreme values. On the other hand, in a complex, changing system, old information may become obsolete as fast as new information is gathered; as a result, there may be an upper limit on how much can be known about $\mathbf{p}(\mathbf{x})$. Informally, if we never have more than 100 valid, current observations, we can never learn much about the 99^{th} percentile of $\mathbf{p}(\mathbf{x})$. With a small number of observations, the best available estimate of $\mathbf{p}(\mathbf{x})$ is a Student's t or other fat-tailed distribution, with relatively high probabilities of extreme values.

There are plausible damage functions, such as $\mathbf{D}(\mathbf{x}) = \mathbf{b}e^{\mathbf{cx}}$ (with $\mathbf{c} > 0$), for which the integral in (3) converges if $\mathbf{p}(\mathbf{x})$ is a normal distribution, but diverges, or tends toward infinity, if $\mathbf{p}(\mathbf{x})$ is a Student's t distribution. Weitzman's "dismal theorem" formalizes and generalizes this notion, proving that in cases of limited information about unlimited risks, the expected value of damages is infinite – due to the irreducible probabilities of worst-case outcomes (Weitzman 2009). In practice, the infinite expected value of damages should be detectable by a Monte Carlo analysis with a very large number of runs: the calculated average value of damages should become ever larger as the number of runs increases, reflecting the weight of the occasional draws of parameters farther and farther out on the fat tail of the distribution. The damages associated with those extreme parameter values should grow large more rapidly than they become rare, driving the average steadily upward as the number of runs increases.

In terms of climate policy, cost-benefit analysis implies that the expected value of damages in (3), per ton of carbon, is the amount that should be spent, at the margin, to reduce emissions. If that value is infinite, detailed cost-benefit calculation becomes pointless, and nothing is as important as reduction in emissions.

4. The shape of the damage function

The Stern Review (Stern 2006) represented a break with earlier climate economics modeling in several respects. The most widely discussed innovation was Stern's low discount rate, which greatly increases the importance of future climate damages (among many others, Ackerman 2009; Nordhaus 2007b). Of comparable importance is Stern's treatment of uncertainty, which also causes a marked increase in the present value of future damages.

The PAGE model, used in the Stern Review, incorporates an estimate of catastrophic risk, with the magnitude of potential catastrophe based on the work of Nordhaus. As with DICE, the catastrophe becomes possible at a temperature threshold, and becomes more likely as temperatures rise above the threshold. In PAGE, however, the catastrophe is modeled through a Monte Carlo analysis, not a certainty-equivalent cost estimate; catastrophic costs are calculated separately from ordinary damages, not subsumed into an aggregate damage function (Hope 2006).

PAGE is far from the last word in climate economics modeling. Questions have been raised about whether its default input data lead to serious underestimates of climate damages (Ackerman *et al.* 2009b; Baer 2007). On the other hand, the PAGE damage estimates are higher than those produced by many other models; sensitivity analyses have shown that the Monte Carlo approach sharply increases the PAGE estimates, since the few runs with extreme parameter values have a big effect on average outcomes (Dietz *et al.* 2007).

PAGE makes the damage function exponent, **N** in equation (1), a Monte Carlo parameter using a triangular distribution with minimum 1.0, mode 1.3, and maximum 3.0; this raises the damage estimates compared to a fixed exponent of 2. Even though the mean value of **N** is only 1.7, the few runs with values closer to 3 have a large effect on the average.

A sensitivity analysis on Stern's results found that fixing the exponent at 3 would increase Stern's estimate of global damages by 23 percent of world output (Dietz *et al.* 2007). Since there is no empirical evidence that the exponent is *not* 3, this should be considered one of the important and provocative quantitative results of the Stern analysis, emphasizing the extent of uncertainty in damage estimates and the critical role of the virtually unobservable damage function exponent.

5. Our experiment

To test the importance of these ideas we modified DICE-2007, to treat the damage function exponent and the climate sensitivity parameter as random variables.

5.1. Climate sensitivity. In his discussion of uncertainty in the climate sensitivity parameter, Weitzman cites several IPCC estimates as well as his own extrapolations.[4] According to the 2007 IPCC assessment (as cited in Weitzman 2009), the central estimate for climate sensitivity is 3; the value is likely to be between 2 and 4.5, and very likely to be above 1.5. In IPCC terminology, "likely" means a two-thirds probability, while "very

[4] For use in DICE, the relevant estimate is Weitzman's S_1, the direct effect without the longer-term, indirect feedback; Weitzman argues that the ultimate effect S_2 is roughly twice as large. The technical case for a long-term climate sensitivity twice as large as the IPCC estimates is discussed, for instance, in Hansen et al. (2008).

likely" means a 90 percent probability. So the IPCC estimates imply that the 10^{th} percentile value for climate sensitivity is 1.5; the 17^{th} percentile is 2; the 50^{th} percentile is 3; and the 83^{rd} percentile is 4.5. Weitzman adds his own estimates that the corresponding 95^{th} percentile value is 7, and the 99^{th} percentile is 10.

A lognormal probability distribution provides a very good fit to these estimates.[5] The cumulative distribution, with the IPCC and Weitzman data points included as large dots, is shown in Figure 1, and the corresponding probability distribution is shown in Figure 2. The underlying normal distribution of the log of the variable has a mean of 1.071 and a standard deviation of 0.527. The lognormal distribution itself has a mean of 3.352, and a standard deviation of 1.896. We use this lognormal distribution for the climate sensitivity parameter in our Monte Carlo analysis.

Figure 1

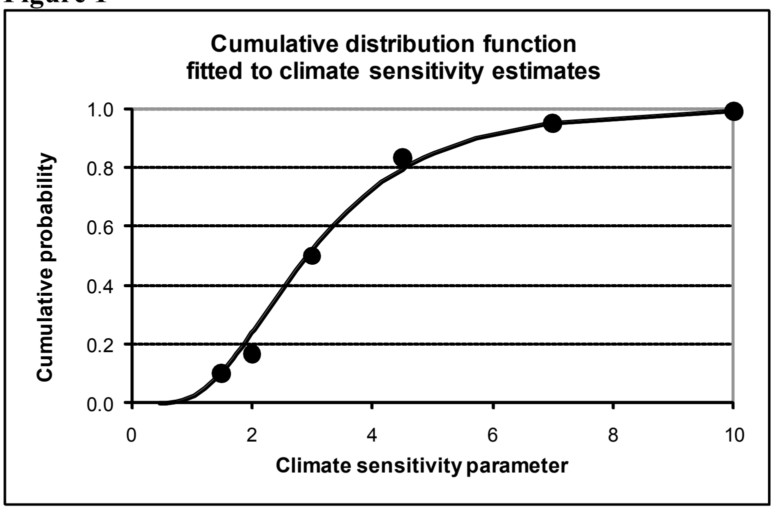
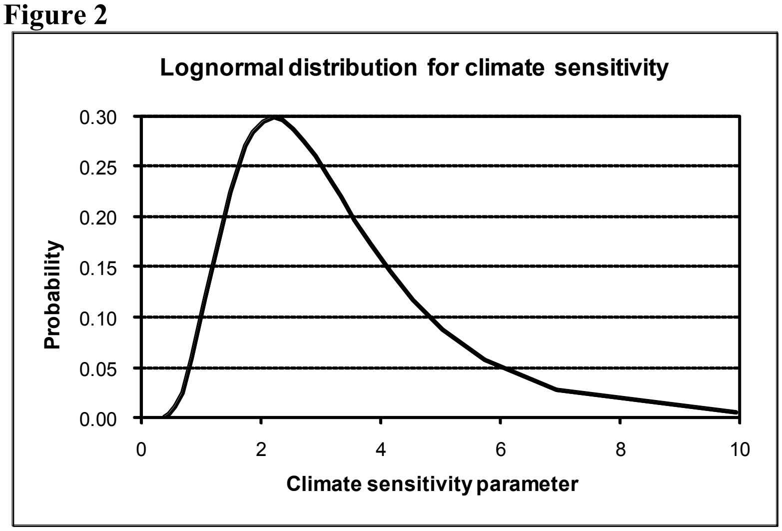


Figure 2

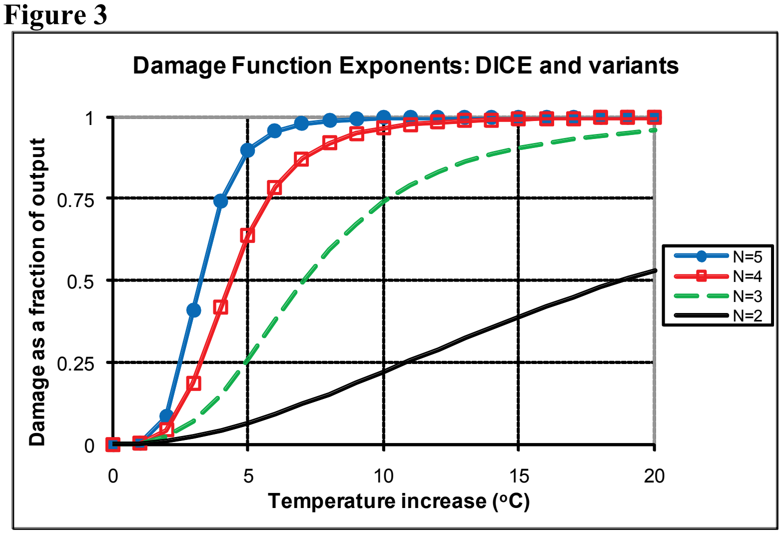


[5] The curve was fitted to minimize the sum of squared errors at the six point estimates shown in Figure 1.

5.2. The damage function exponent. As noted above, DICE uses equation (2) to model damages, with **N** = 2 and **a** = .002838. This appears to be purely a mathematical convenience. We are not aware of any empirical support for relationships such as equations (1) or (2), even at historical temperatures – let alone for "out of sample" forecasting of damages at temperatures beyond the historical range, which is what really matters. In particular, there is no obvious source of support for the crucial assumption that **N** = 2.

The exponent **N** measures the speed with which damages increase as temperatures rise. Figure 3 graphs equation (2), the DICE damage function, holding **a** constant, for **N** = 2, 3, 4, and 5. Damages rise at a leisurely pace for **N** = 2, with less than half of world output destroyed by climate change until **T** = 19°C – which appears to be well beyond the temperature needed to cause the end of human life as we know it. In contrast, as **N** rises, half of world output is lost to climate change at temperatures of about 7°C for **N** = 3; 4.5°C for **N** = 4; or 3.5°C for **N** = 5. If equation (2) is used with the Nordhaus value of **a**, then **N** in the range of 3 to 5 is needed to match the sense of urgency in the current discussion of climate science.

Figure 3



As **N** approaches infinity, equation (2) approaches a vertical line. This would be the appropriate shape for the damage function under the hypothesis that there is a threshold for an abrupt world-ending (or at least economy-ending) discontinuity, while damages below that threshold are so small that they can be ignored by comparison.

Thus choosing a larger **N** ("closer to infinity") means moving closer to the view that complete catastrophe sets in at some finite temperature threshold. Choosing a smaller **N** means emphasizing the gradual rise of damages rather than the risk of discontinuous, catastrophic change.

In our Monte Carlo analysis we used equation (2), allowing **N** to vary from a minimum of 1 to a maximum of 5, assuming a triangular distribution with the most likely (modal) value at **N** = 2. On the low end, it does not seem plausible to consider **N** < 1; at the other extreme, Figure 3 suggests that the damage curve for **N** = 5 is close enough to vertical to reflect a substantial risk of catastrophe.

5.3. Research methods. We made a minor software modification, to allow DICE to run in a Monte Carlo mode, reading in new parameter values, running the model, saving selected output, and repeating. We used @RISK[6], a commercially distributed Monte Carlo software package, to generate random values for the climate sensitivity parameter and the damage function exponent, drawn from the probability distributions described above.

Our only changes to DICE itself, other than the Monte Carlo analysis on the two parameters, were the removal of the ceiling on temperature increases and the floor under the capital stock; the latter effectively implies a floor under per capita consumption. These ad hoc features of DICE artificially prevent forecasts of extreme outcomes, although neither is a binding constraint in the DICE default business-as-usual or optimal policy forecasts. In all other respects, we used the 2007 version of the DICE software and default data sets.

We performed a series of Monte Carlo analyses of DICE ranging from 1,000 to 500,000 runs. For each run we drew a climate sensitivity parameter from the lognormal distribution shown in Figure 2, and a damage function exponent from the triangular distribution described above. The huge number of iterations was motivated by curiosity about the effects of the tails of the distributions, particularly for the climate sensitivity parameter. (Although inspired in part by Weitzman's analysis of uncertainty, our study is not a test of his theories: the DICE damage function does not lead to an infinite value for the integral in equation (3), even with a fat-tailed probability distribution.)

The burden of calculation for this analysis was potentially overwhelming; if carried out in a straightforward manner, it would have required running DICE 500,000 times. After running Monte Carlo analyses with tens of thousands of iterations, we switched to a discrete approximation, analogous to the "finite element method" used in engineering to obtain numerical solutions to complex systems of equations. This approximation made it possible to push the effective number of iterations even higher. Specifically, we rounded each randomly drawn value for the climate sensitivity parameter to the nearest integer, and rounded each damage exponent to the nearest multiple of 0.25. DICE is continuous in both parameters throughout the range we explored, so very little information is lost by this procedure.[7] As a result, we could associate each parameter pair with a point on a two-dimensional grid: since the largest climate sensitivity in any of our samples was 35, and damage exponents ranged from 1 to 5 in steps of 0.25, every parameter pair was approximated by one of the 35 * 17 = 595 grid points. Hence only 595 runs of DICE

[6] @RISK v4.5 (2005), Palisade Corporation, Ithaca, NY, http://www.palisade.com
[7] We found that DICE behaves erratically with a climate sensitivity parameter below about 0.6. Therefore we rounded all climate sensitivity estimates between 0 and 1 up to 1. This affects a very small fraction of the samples: the probability that the climate sensitivity parameter is less than 0.5 (so that it would not naturally round to 1) is 0.0004.

were sufficient to approximate the outcomes for each parameter pair, even for our largest samples.

To arrive at the results presented below, we drew random samples of up to 500,000 instances for each parameter, and then, for each parameter pair, used the model results at the closest of the grid points. The expected values of model results presented below are the averages of those grid-point approximations. For the smaller samples, where we also ran individual calculations for each parameter pair, we confirmed that the discrete approximation produces results that are very close to the exact values.

6. Results

6.1. Measures of economic catastrophe. DICE is designed to maximize welfare, or utility, which, in the 2007 version of the model, is a linear function of the inverse of per capita consumption.[8] The present value of utility in the business-as-usual scenario is an interesting but limited measure of economic impacts of climate change. It is hard to interpret because it is not expressed in any natural or familiar units: how much of a welfare loss represents a catastrophe, as opposed to a minor downturn? Moreover, the present value of utility over six centuries is being maximized; the result is shaped by the discount rate, determining the relative weights of future vs. present welfare. (For this analysis we made no changes to the DICE discount rate.[9]) In light of these problems, we also used two other, more intuitive measures of economic performance.

One measure is the minimum level of per capita consumption reached at any time during the 600 years of the business-as-usual scenario. The DICE default projection is that despite climate damages, per capita incomes are monotonically increasing. (PAGE, the Stern Review's model, also projects continuous growth throughout its multi-century forecasts.) Climate damages and climate policies have some effect on the rate of economic growth, but for small perturbations of the DICE defaults, the growth rate always remains positive. In such cases, the minimum per capita consumption for the DICE business-as-usual scenario is the value in the initial year, a worldwide average of about $6,600.[10] On the other hand, if climate damages become severe enough, growth rates will turn negative, and eventually incomes and consumption in later years will drop below the initial levels. The lower the scenario minimum per capita consumption falls, the worse the economic impact of climate change has become.

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[8] In the standard IAM formulation, if $\mathbf{C}$ is per capita consumption and $\eta > 1$ is the income elasticity of the marginal utility of consumption, then welfare is proportional to $-\mathbf{C}^{1-\eta}$. DICE-2007 assumes $\eta = 2$, implying that welfare is proportional to $-\mathbf{C}^{-1}$. Since the units of welfare are arbitrary, DICE applies a linear transformation, reporting welfare = $a - b\mathbf{C}^{-1}$.

[9] A parallel calculation using the Stern Review's discount rate yielded qualitatively similar, although quantitatively different, results. For simplicity of exposition, only the results with the DICE discount rate are presented here.

[10] An artifact of our calculations is that, for $\mathbf{N} > 2$, damages are lower and incomes are higher than in the DICE base case until the temperature increase reaches 1^{o} (see equation (2)). So for high exponents, initial damages are lower and initial consumption is higher than in the DICE defaults. Initial per capita consumption levels range from about $6600 to $7000 across our grid of DICE runs.

Our second measure is the time required to reach complete abatement of carbon emissions in the optimal (welfare-maximizing) policy scenario. DICE assumes that any degree of abatement, up to and including 100 percent reduction in carbon emissions, is available in any year. At any moment in time, costs rise steeply as the percentage reduction in emissions approaches 100 percent; over time the cost of any level of emission reduction gradually declines. It would be possible to eliminate all carbon emissions in the first time period, at an assumed cost of about 5.2 percent of world output. However, using DICE default values, the optimal reduction path does not reach 100 percent abatement until 200 years have passed. In scenarios with greater climate damages, it becomes desirable to phase out emissions more quickly. The more serious the economic consequences of climate change become, the shorter the time required for complete abatement on the optimal path.

6.2. Monte Carlo analysis: summary results. Our results for the whole sample showed a reasonable match to the DICE defaults, and remarkably little variation with sample size, as shown in Table 1. The first two columns of results – the sample averages for the present value of total utility and for the minimum per capita consumption – are for the business-as-usual scenario. The last column, the sample average for the decades to reach complete abatement, is for the corresponding optimal policy scenario.

In other experiments (not shown here), we fixed the damage function exponent at 2, and then at 5, allowing only the climate sensitivity parameter to vary. In both cases, the results likewise showed no significant changes with sample size.

Table 1

Monte Carlo analysis results

Climate sensitivity drawn from lognormal distribution (Figure 2 above).
Damage function exponent drawn from triangular distribution: min=1, mode=2, max=5.

Sample size	*PV of scenario total utility*	*Minimum per capita consumption*	*Decades to complete abatement*
1,000	139,700	$6,590	17.9
10,000	140,300	$6,610	18.0
50,000	140,700	$6,610	18.0
100,000	140,600	$6,610	18.0
200,000	140,500	$6,610	18.0
500,000	140,500	$6,610	18.0
DICE defaults	149,800	$6,620	20

6.3. Mapping the grid. To understand this surprising pattern of results, it may be helpful to examine the grid of outcomes used in our calculations. For our three outcomes measures, Figures 4, 5, and 6 present three-dimensional graphs, with the outcome on the vertical axis, and the climate sensitivity parameter and damage function exponent on the horizontal axes.[11] In each graph, outcomes become precipitously worse when moving toward the lower front corner, i.e. increasing both parameters. In contrast, the upper back corner, representing low values of both parameters, shows consistently better outcomes. The DICE defaults are represented by the circular dot in each graph, relatively close to the upper back corner.

The graphs present only a portion of our parameter grid; they are truncated at a climate sensitivity parameter of 20 because almost nothing qualitatively different occurs beyond that point. That is, by the time climate sensitivity reaches 20, the results have become about as bad as they are going to get. This represents a point quite far down the tail of the probability distribution; the probability of exceeding 20 is 0.00013, or about 1 in 8,000.

Figure 4 presents the graph of utility in the business-as-usual scenario (measured in arbitrary units of utility, with an arbitrary constant added for convenience in graphing; see footnote 7). The DICE default values (the large dot) are located well within a region in parameter space where the present value of utility is high and relatively invariant. As both parameters increase, utility eventually plunges downward.

Figure 4

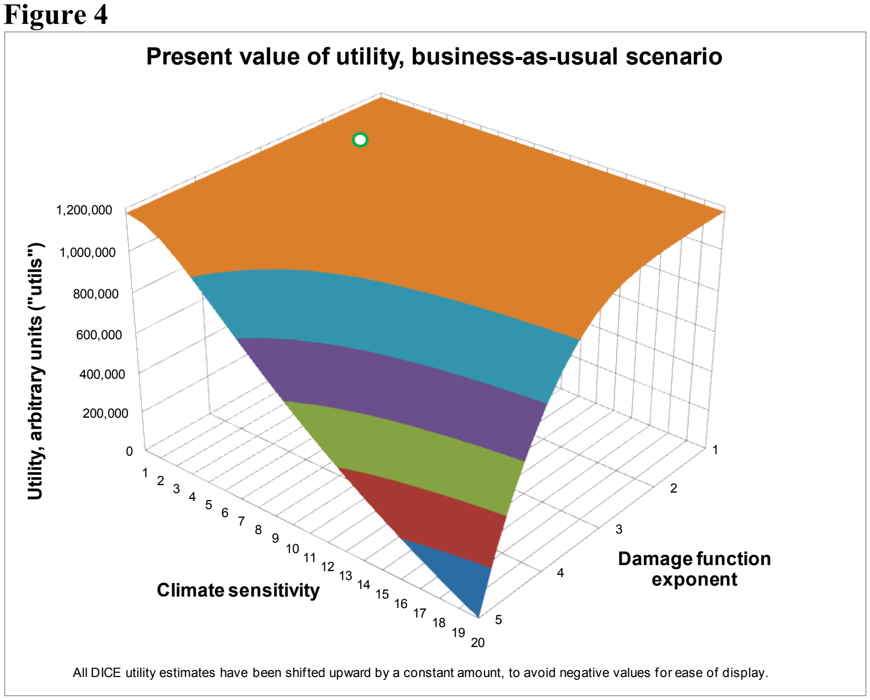
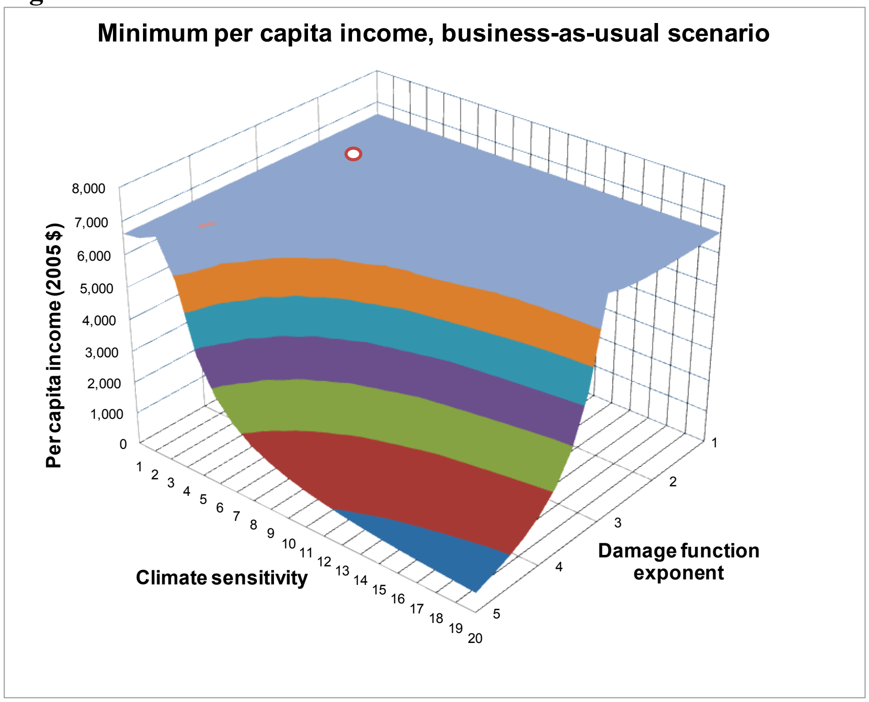


Figure 5 presents a similar graph of the minimum level of per capita consumption that occurs in the business-as-usual scenario. The large, nearly flat area toward the upper back of the graph represents cases in which climate damages never drive per capita incomes

[11] The perspective is the same in all three graphs: the viewer is looking toward the origin (which is hidden behind the graphed surface) from a point high in the positive orthant – i.e., from a high, positive value for all three axis variables.

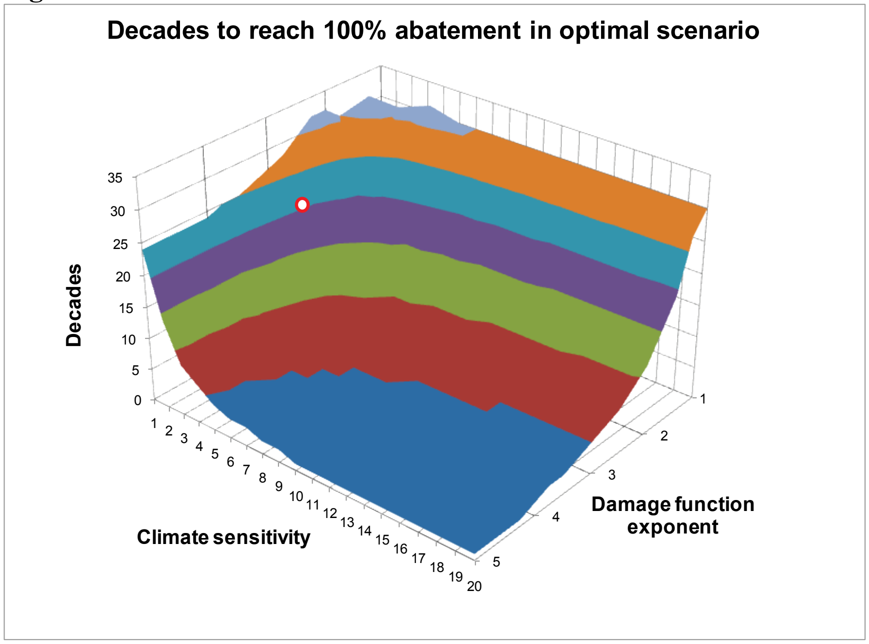
below the initial value. The DICE defaults are again well within the high plateau of happy outcomes, while the terrain slopes rapidly downward as both parameters rise toward the lower front corner. The lowest points shown here represent drastic, potentially unsustainable losses of income and consumption due to climate damages.

Figure 5



A somewhat different picture is presented in Figure 6, showing the number of decades required to reach 100 percent abatement in the optimal policy scenario. By this measure, there is no plateau of constant outcomes; the optimal path to decarbonization is a leisurely, multi-century stroll at low values of both parameters, but becomes a more and more rapid dash as the parameters rise toward the front of the graph. At the DICE defaults (again, the circular dot on the graph), 100 percent abatement does not occur for 200 years; at the highest parameter values shown here, it occurs in the model's first decade.

Figure 6



In light of these graphs, the explanation of our nearly invariant Monte Carlo results, in Table 1, is that those results are probability-weighted averages across the entire parameter space. The good outcomes in the low-parameter region of the map have high probability and dominate the averages. The averages conceal the fact that outcomes become much worse as both parameters increase. The DICE treatment of climate and economic processes does not allow outcomes to worsen rapidly enough to cause the Weitzman effect, i.e. an infinite expected value of loss. In DICE, the risk of both parameters increasing at once is not infinitely bad for economic welfare – just very bad.

6.4. Credible worst cases. While Figures 4, 5, and 6 help to visualize the parameter space of the DICE model, they do not display our assumptions about the probability distributions for the two parameters. The graphs' upper limit of 20 for climate sensitivity is reached or exceeded, as mentioned earlier, with a probability of about 1 in 8,000. Thus one could argue that the figures implicitly make the unfair suggestion that very unlikely values should be given equal credence with much more likely ones.

Our probability distributions for the two parameters have differing foundations. The climate sensitivity parameter is the subject of significant empirical research; while there is limited information available, leading to a fat-tailed distribution, this is not a case of arbitrary or fact-free assignment of probabilities. Unfortunately, "arbitrary" and "fact-free" are reasonable characterizations of the distribution we used for the damage function exponent – and our work is not at all unique on this point. There is essentially no relevant empirical research, and it is not clear whether there ever could be any, except after the fact. Our assumed distribution was selected purely for comparability with guesses made by other analysts.

Our final look at the data focuses on what might be considered credible worst cases for climate sensitivity, and considers the implication of different damage function exponents. Recall that the 50th percentile for climate sensitivity is 3, and the 99th percentile is 10. The climate changes of the twenty-first century are an experiment with immense stakes, which will only happen once; in the absence of better information, it is surely worth considering what risks up to the 99th percentile would look like. To that end, Figures 7, 8, and 9 show how our three measures of economic outcomes change as climate sensitivity rises from 3 to 10, at damage function exponents of 2, 3, 4, and 5.

Figure 7 graphs the relationship between climate sensitivity and the present value of total scenario utility, in the business-as-usual scenario. (Again, the units are arbitrary.) At a damage function exponent of 2 or 3, utility is nearly invariant across this range. At an exponent of 4, and even more so at 5, utility is strongly related to climate sensitivity. In short, growing climate sensitivity, implying worsening climate outcomes, hardly matters to DICE, with the default exponent of 2; it is barely beginning to matter at 3. To support the widespread notion that climate sensitivity and climate outcomes *must* be of great importance, an exponent of 4 or 5 is needed.

Figure 7

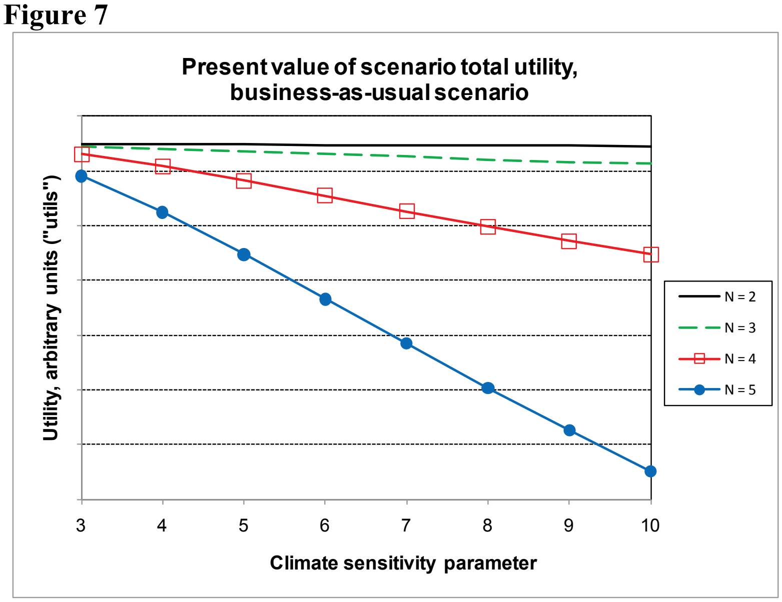
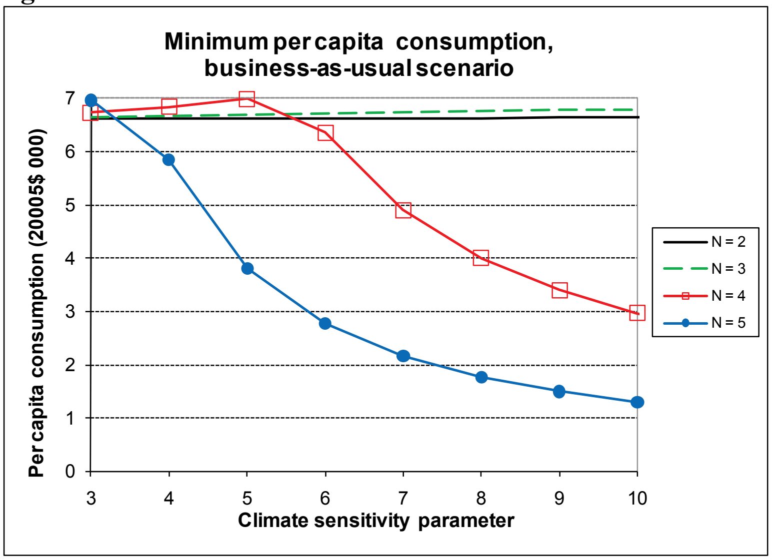


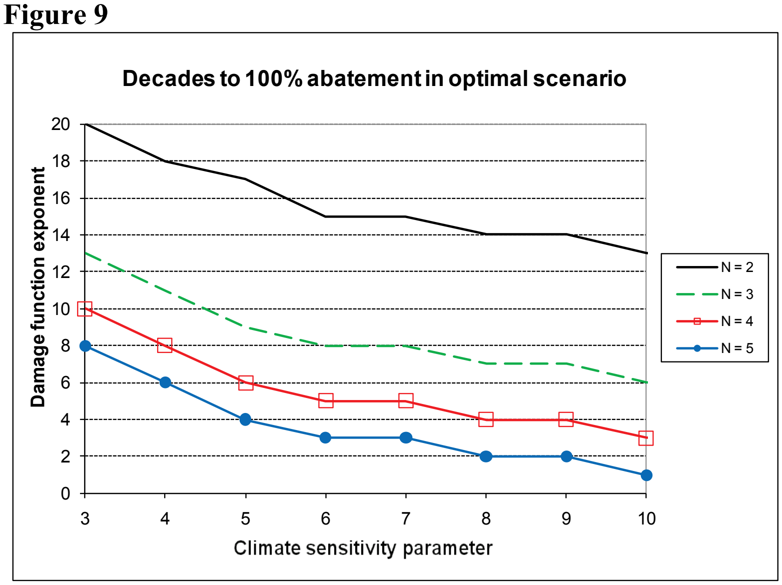
Figure 8 tells essentially the same story, in terms of the business-as-usual scenario minimum per capita consumption. At a damage function exponent of 2 or 3, climate damages never drive per capita consumption below the initial value, so long as climate sensitivity stays below 10. On the other hand, minimum per capita consumption begins falling midway through this range of climate sensitivity with an exponent of 4, and throughout the range with an exponent of 5. Again, a credible range of worst-case values for climate sensitivity yields a dramatic worsening of economic outcomes at higher exponents, while leaving the baseline conditions more or less unchanged at exponents of 2 or 3.

Figure 8



The story is different in terms of the optimal time to reach complete abatement, as shown in Figure 9. Increases in climate sensitivity accelerate the abatement process, regardless of the damage function exponent; indeed, the lines on the graph move in lockstep, with the greatest acceleration of abatement occurring between climate sensitivity values of 3 and 6. The difference in urgency expressed by the different exponents can be read in the values (of the vertical coordinates) shown in Figure 9. At the DICE default exponent of 2, the optimal path to complete abatement takes two centuries at a climate sensitivity of 3, and still needs more than one century at climate sensitivity of 10. At an exponent of 4 or 5, complete abatement occurs in a century or less at climate sensitivity of 3, and in 30 years or less at climate sensitivity of 10. Again, the sense of urgency about reducing carbon emissions in the next few decades is endorsed by DICE with an exponent of 4 or 5, at climate sensitivity values well below the 99th percentile.

Figure 9



7. Summary

The real-world urgency of the climate crisis, and the threat of physically catastrophic outcomes that motivates immediate action, can be reflected in the traditionally optimistic DICE model, with just two changes. If the climate sensitivity parameter turns out to be well above 3, and the damage function exponent is 4 or 5, then business-as-usual utility and minimum consumption levels collapse, and the optimal policy involves rapid elimination of carbon emissions. At an exponent of 2 or even 3, in contrast, dangerously higher climate sensitivity inspires DICE to offer only a modest acceleration of its leisurely default path to decarbonization, but barely perturbs total utility or minimum per capita consumption. In short, if it is a boundary constraint that an IAM should be qualitatively consistent with the much better established science of climate change, then DICE needs a damage function exponent of at least 4 or 5 – or other major modifications.

Our study found that a fat-tailed probability distribution for climate sensitivity alone does not have much effect on DICE projections. If either the damage function exponent remains at or near the default value of 2, or climate sensitivity remains at or near the default value of 3, then DICE projects relatively little economic harm. With plausible changes in both parameters, however, DICE forecasts disastrous economic decline and calls for rapid mitigation.

The bad news is that the optimal policy recommended by a standard IAM such as DICE is completely dependent on the choice of key, uncertain parameters. The good news is that there is no reason to believe that sound economics, or even the choice of established, orthodox models, creates any grounds for belittling the urgency of the climate crisis.

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